Rettendon Primary School

How we teach calculations:

Calculation Policy for Mathematics

September 2014

About our Calculation Policy

The following calculation policy has been devised to meet requirements of the National Curriculum 2014 for the teaching and learning of mathematics, and is also designed to give pupils a consistent and smooth progression of learning in calculations across the school. Please note that early learning in number and calculation in Reception follows the Development Matters EYFS document, and this calculation policy is designed to build on progressively from the content and methods established in the Early Years Foundation Stage.

Age stage expectations

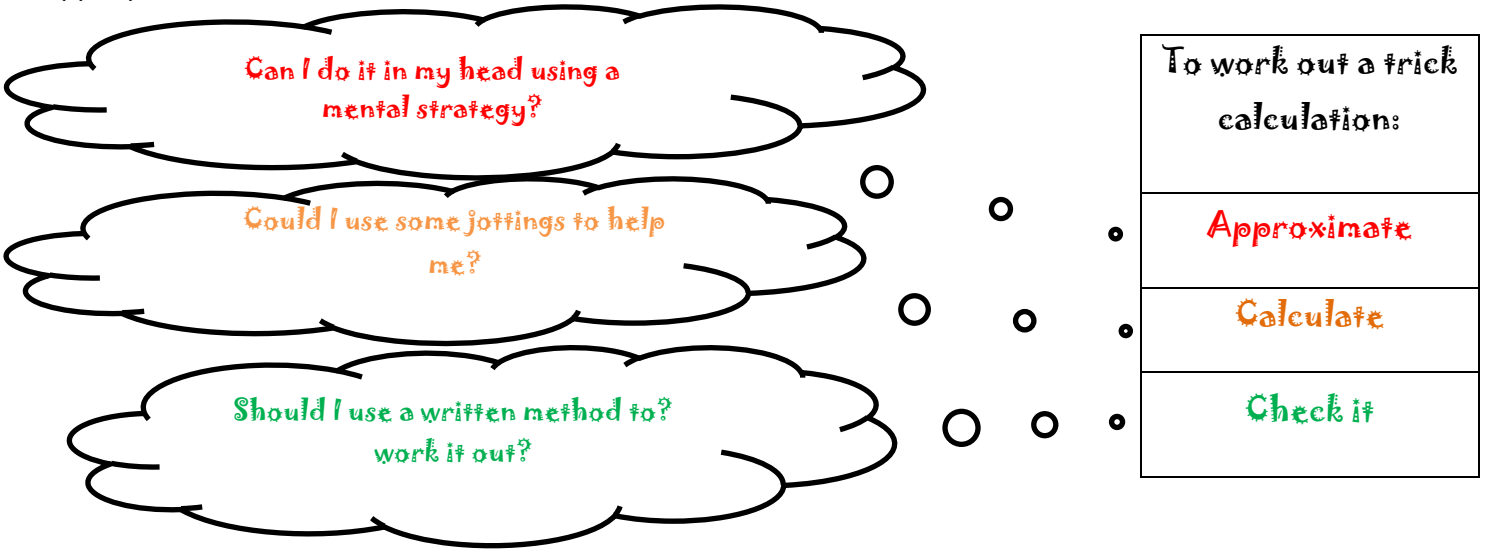
The calculation policy is organised according to age stage expectations as set out in the National Curriculum 2014, **however at Rettendon we strive to extend our children and they are set according to their ability and potential**, being moved onto the next level when we feel they are ready.

Providing a context for calculation:

It is important that any type of calculation is given a real life context or problem solving approach to help build children's understanding of the purpose of calculation, and to help them recognise when to use certain operations and methods when faced with problems. This must be a priority within calculation lessons.

Choosing a calculation method:

Children will be taught are variety of methods and encouraged to use the following processes in deciding what approach they will take to a calculation, to ensure they select the most appropriate method for the numbers involved:



Early Maths

Research on children's learning in the first six years of life demonstrates the importance of early experiences in mathematics. An engaging and encouraging climate for children's early encounters with mathematics develops their confidence in their ability to understand and use mathematics. These positive experiences help children to develop dispositions such as curiosity, imagination, flexibility, inventiveness, and persistence, which contribute to their future success in and out of school (Clements & Conference Working Group, 2004).

The NCTM (National Council of Teachers of Mathematics) states

"Young learners' future understanding of mathematics requires an early foundation based on a high-quality, challenging, and accessible mathematics education. Young children in every setting should experience mathematics through effective, research-based curricula and teaching practices. Such practices in turn require that teachers have the support of policies and resources that enable them to succeed in this challenging and important work."

They go on to highlight how early maths can support the aims of the new Curriculum 2014:

"Early childhood educators should actively introduce mathematical concepts, methods, and language through a variety of appropriate experiences. Teachers should guide children in seeing connections of ideas within mathematics as well as with other subjects, developing their mathematical knowledge throughout the day and across the curriculum. They must encourage children to communicate, explaining their thinking as they interact with important mathematics in deep and sustained ways."

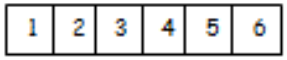
THE EARLY YEARS FOUNDATION STAGE

Mathematics involves providing children with opportunities to develop and improve their skills in counting, understanding and using numbers, calculating simple addition and subtraction problems; and to describe shapes, spaces, and measures.

(Statutory Framework for the Early Years Foundation Stage, DfE: 2012)

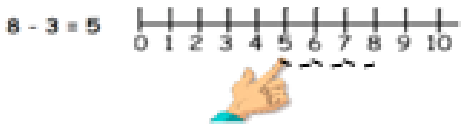
EARLY YEARS - ADDITION

Maths for young children should be meaningful. Where possible, concepts should be taught in the context of real life.

GUIDANCE / MODELS AND IMAGES	KEY VOCABULARY
If available, Numicon shapes are introduced straight away and can be used to: • identify 1 more/less • combine pieces to add. • find number bonds. • add without counting.  Children can record this by printing or drawing around Numicon pieces. Children begin to combine groups of objects using concrete apparatus  Construct number sentences verbally or using cards to go with practical activities. Children are encouraged to read number sentences aloud in different ways "Three add two equals 5" "5 is equal to three and two" Children make a record in pictures, words or symbols of addition activities already carried out. Solve simple problems using fingers  5 + 1 = 6 Number tracks can be introduced to count up on and to find one more:  What is 1 more than 4? 1 more than 13? Number lines can then be used alongside number tracks and practical apparatus to solve addition calculations and word problems.  **Children will need opportunities to look at and talk about different models and images as they move between representations.**	**Games and songs can be a useful way to begin using vocabulary involved in addition e.g.** Alice the Camel add more and make sum total altogether score double one more, two more, ten more… how many more to make…? how many more is… than…?

EARLY YEARS - SUBTRACTION

Maths for young children should be meaningful. Where possible, concepts should be taught in the context of real life.

GUIDANCE / MODELS AND IMAGES	KEY VOCABULARY
Children begin with mostly pictorial representations XXX X X Concrete apparatus is used to relate subtraction to taking away and counting how many objects are left. Concrete apparatus models the subtraction of 2 objects from a set of 5. Construct number sentences verbally or using cards to go with practical activities. Children are encouraged to read number sentences aloud in different ways "five subtract one leaves four" "four is equal to five subtract one" Children make a record in pictures, words or symbols of subtraction activities already carried out. Solve simple problems using fingers  5 - 1 = 4 Number tracks can be introduced to count back and to find one less:  What is 1 less than 9? 1 less than 20? Number lines can then be used alongside number tracks and practical apparatus to solve subtraction calculations and word problems. Children count back under the number line.  **Children will need opportunities to look at and talk about different models and images as they move between representations.**	**Games and songs can be a useful way to begin using vocabulary involved in subtraction e.g.** Five little men in a flying saucer take (away) leave how many are left/left over? how many have gone? one less, two less... ten less... how many fewer is... than...? difference between is the same as

EARLY YEARS - MULTIPLICATION

Maths for young children should be meaningful. Where possible, concepts should be taught in the context of real life

GUIDANCE / MODELS AND IMAGES	KEY VOCABULARY
The link between addition and multiplication can be introduced through doubling. If available, Numicon is used to visualise the repeated adding of the same number. These can then be drawn around or printed as a way of recording. Children begin with mostly pictorial representations:   How many groups of 2 are there? Real life contexts and use of practical equipment to <u>count in repeated groups of the same size</u>:  How many wheels are there altogether?  How much money do I have? Count in twos; fives; tens both aloud and with  objects Children are <u>given multiplication problems set in a real life context</u>. Children are encouraged to visualise the problem. How many fingers on two hands? How many sides on three triangles? How many legs on four ducks? Children are encouraged to read number sentences aloud in different ways "five times two makes ten" "ten is equal to five multiplied by two"	lots of groups of times multiply multiplied by multiple of once, twice, three times... ten times... ...times as (big, long, wide... and so on) repeated addition double

EARLY YEARS - DIVISION and FRACTIONS

Maths for young children should be meaningful. Where possible, concepts should be taught in the context of real life.

GUIDANCE / MODELS AND IMAGES	KEY VOCABULARY
The ELG states that children **solve problems, including doubling, halving and sharing.** Children need to see and hear representations of division as both grouping and sharing. Division can be introduced through halving. Children begin with mostly pictorial representations linked to real life contexts:  **Grouping model** Mum has 6 socks. She grouped them into pairs – how many pairs did she make?  **Sharing model** I have 10 sweets. I want to share them with my friend. How many will we have each? Children have a go at recording the calculation that has been carried out.	halve share, share equally one each, two each, three each… group in pairs, threes… tens equal groups of divide divided by divided into left, left over

FRACTIONS

GUIDANCE / MODELS AND IMAGES	KEY VOCABULARY
Although not explicit in the Development Matters document, the sharing model is a useful way of introducing young children to fractions and calculating with fractions. Setting the problems in real life context and solving them with <u>concrete apparatus</u> will support children's understanding. "I have got 5 bones to share between my two dogs. How many bones will they get each?"  Children have a go at recording the calculation that has been carried out. 2 ½ + 2 ½ = 5	As division vocabulary plus: fraction half halves third thirds

Development Matters in the Early Years Foundation Stage (EYFS)

This non-statutory guidance material supports practitioners in implementing the statutory requirements of the EYFS.

Links to calculation:

22 – 36 months	30 – 50 months	40 – 60 months	**Early Learning Goal for Numbers**
Creates and experiments with symbols and marks representing ideas of number. Begins to make comparisons between quantities. Uses some language of quantities, such as *'more'* and *'a lot'.* Knows that a group of things changes in quantity when something is added or taken away.	Beginning to represent numbers using fingers, marks on paper or pictures. Compares two groups of objects, saying when they have the same number. Separates a group of three or four objects in different ways, beginning to recognise that the total is still the same.	Says the number that is one more than a given number. Finds one more or one less from a group of up to five objects, then ten objects. In practical activities and discussion, beginning to use the vocabulary involved in adding and subtracting. Records, using marks that they can interpret and explain.	**Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number.** **Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer.** **They solve problems, including doubling, halving and sharing.**

Mathematics: Numbers

	A Unique Child: observing what a child is learning	Positive Relationships: what adults could do	Enabling Environments: what adults could provide
 Birth - 11 months	• Notices changes in number of objects/images or sounds in group of up to 3.	• Sing number rhymes as you dress or change babies, e.g. 'One, Two, Buckle My Shoe'. • Move with babies to the rhythm patterns in familiar songs and rhymes. • Encourage babies to join in tapping and clapping along to simple rhythms.	• Display favourite things so that a young baby can see them. • Provide a small group of the same objects in treasure baskets, as well as single items, e.g. two fir cones or three shells. • Create a mobile, occasionally changing the number of items you hang on it. • Collect number rhymes which are repetitive and are related to children's actions and experiences, for example, 'Peter Hammers with One Hammer'. • Use song and rhymes during personal routines, e.g. 'Two Little Eyes to Look Around', pointing to their eyes, one by one. • Collect number and counting rhymes from a range of cultures and in other languages. This will benefit all children and will give additional support for children learning English as an additional language.
 8-20 months	• Develops an awareness of number names through their enjoyment of action rhymes and songs that relate to their experience of numbers. • Has some understanding that things exist, even when out of sight.		
 16-26 months	• Knows that things exist, even when out of sight. • Beginning to organise and categorise objects, e.g. putting all the teddy bears together or teddies and cars in separate piles. • Says some counting words randomly.	• Use number words in meaningful contexts, e.g. *'Here is your other mitten. Now we have two'.* • Talk to young children about 'lots' and 'few' as they play. • Talk about young children's choices and, where appropriate, demonstrate how counting helps us to find out how many. • Talk about the maths in everyday situations, e.g. doing up a coat, one hole for each button. • Tell parents about all the ways children learn about numbers in your setting. Have interpreter support or translated materials to support children and families learning English as an additional language	• Provide varied opportunities to explore *'lots'* and *'few'* in play. • Equip the role-play area with things that can be sorted in different ways. • Provide collections of objects that can be sorted and matched in various ways. • Provide resources that support children in making one-to-one correspondences, e.g. giving each dolly a cup.
 22-36 months	• Selects a small number of objects from a group when asked, for example, *'please give me one', 'please give me two'.* • Recites some number names in sequence. • Creates and experiments with symbols and marks representing ideas of number. • Begins to make comparisons between quantities. • Uses some language of quantities, such as *'more'* and *'a lot'.* • Knows that a group of things changes in quantity when something is added or taken away.	• Encourage parents of children learning English as an additional language to talk in their home language about quantities and numbers. • Sing counting songs and rhymes which help to develop children's understanding of number, such as 'Two Little Dickie Birds'. • Play games which relate to number order, addition and subtraction, such as hopscotch and skittles and target games.	• Make a display with the children about their favourite things. Talk about how many like apples, or which of them watches a particular TV programme at home. • Provide props for children to act out counting songs and rhymes. • Provide games and equipment that offer opportunities for counting, such as skittles. • Plan to incorporate a mathematical component in areas such as the sand, water or other play areas.

Mathematics: Numbers

	A Unique Child: observing what a child is learning	Positive Relationships: what adults could do	Enabling Environments: what adults could provide
 30-50 months	• Uses some number names and number language spontaneously. • Uses some number names accurately in play. • Recites numbers in order to 10. • Knows that numbers identify how many objects are in a set. • Beginning to represent numbers using fingers, marks on paper or pictures. • Sometimes matches numeral and quantity correctly. • Shows curiosity about numbers by offering comments or asking questions. • Compares two groups of objects, saying when they have the same number. • Shows an interest in number problems. • Separates a group of three or four objects in different ways, beginning to recognise that the total is still the same. • Shows an interest in numerals in the environment. • Shows an interest in representing numbers. • Realises not only objects, but anything can be counted, including steps, claps or jumps.	• Use number language, e.g. *'one', 'two', 'three', 'lots', 'fewer', 'hundreds', 'how many?'* and *'count'* in a variety of situations. • Support children's developing understanding of abstraction by counting things that are not objects, such as hops, jumps, clicks or claps. • Model counting of objects in a random layout, showing the result is always the same as long as each object is only counted once. • Model and encourage use of mathematical language e.g. asking questions such as 'How many saucepans will fit on the shelf?' • Help children to understand that one thing can be shared by number of pieces, e.g. a pizza. • As you read number stories or rhymes, ask e.g. *'When one more frog jumps in, how many will there be in the pool altogether?'* • Use pictures and objects to illustrate counting songs, rhymes and number stories. • Encourage children to use mark-making to support their thinking about numbers and simple problems. • Talk with children about the strategies they are using, e.g. to work out a solution to a simple problem by using fingers or counting aloud.	• Give children a reason to count, e.g. by asking them to select enough wrist bands for three friends to play with the puppets. • Enable children to note the 'missing set', e.g. *'There are none left'* when sharing things out. • Provide number labels for children to use, e.g. by putting a number label on each bike and a corresponding number on each parking space. • Include counting money and change in role-play games. • Create opportunities for children to separate objects into unequal groups as well as equal groups. • Provide story props that children can use in their play, e.g. varieties of fruit and several baskets like Handa's in the story *Handa's Surprise* by Eileen Browne.
 40-60+ months	• Recognise some numerals of personal significance. • Recognises numerals 1 to 5. • Counts up to three or four objects by saying one number name for each item. • Counts actions or objects which cannot be moved. • Counts objects to 10, and beginning to count beyond 10. • Counts out up to six objects from a larger group.	• Encourage estimation, e.g. estimate how many sandwiches to make for the picnic. • Encourage use of mathematical language, e.g. number names to ten: *'Have you got enough to give me three?'* • Ensure that children are involved in making displays, e.g. making their own pictograms of lunch choices. Develop this as a 3D representation using bricks and discuss the most popular choices. • Add numerals to all areas of learning and development, e.g. to a display of a favourite story, such as 'The Three Billy Goats Gruff'.	• Provide collections of interesting things for children to sort, order, count and label in their play. • Display numerals in purposeful contexts, e.g. a sign showing how many children can play on a number track. • Use tactile numeral cards made from sandpaper, velvet or string. • Create opportunities for children to experiment with a number of objects, the written numeral and the written number word. Develop this through matching activities with a range of numbers, numerals and a selection of objects.

Mathematics: Numbers

	A Unique Child: observing what a child is learning	Positive Relationships: what adults could do	Enabling Environments: what adults could provide
	• Selects the correct numeral to represent 1 to 5, then 1 to 10 objects. • Counts an irregular arrangement of up to ten objects. • Estimates how many objects they can see and checks by counting them. • Uses the language of 'more' and 'fewer' to compare two sets of objects. • Finds the total number of items in two groups by counting all of them. • Says the number that is one more than a given number. • Finds one more or one less from a group of up to five objects, then ten objects. • In practical activities and discussion, beginning to use the vocabulary involved in adding and subtracting. • Records, using marks that they can interpret and explain. • Begins to identify own mathematical problems based on own interests and fascinations. **Early Learning Goal** **Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer. They solve problems, including doubling, halving and sharing.**	• Make books about numbers that have meaning for the child such as favourite numbers, birth dates or telephone numbers. • Use rhymes, songs and stories involving counting on and counting back in ones, twos, fives and tens. • Emphasise the empty set and introduce the concept of nothing or zero. • Show interest in how children solve problems and value their different solutions. • Make sure children are secure about the order of numbers before asking what comes after or before each number. • Discuss with children how problems relate to others they have met, and their different solutions. • Talk about the methods children use to answer a problem they have posed, e.g. *'Get one more, and then we will both have two.'* • Encourage children to make up their own story problems for other children to solve. • Encourage children to extend problems, e.g. *"Suppose there were three people to share the bricks between instead of two".* • Use mathematical vocabulary and demonstrate methods of recording, using standard notation where appropriate. • Give children learning English as additional language opportunities to work in their home language to ensure accurate understanding of concepts.	• Use a 100 square to show number patterns. • Encourage children to count the things they see and talk about and use numbers beyond ten • Make number games readily available and teach children how to use them. • Display interesting books about number. • Play games such as hide and seek that involve counting. • Encourage children to record what they have done, e.g. by drawing or tallying. • Use number staircases to show a starting point and how you arrive at another point when something is added or taken away. • Provide a wide range of number resources and encourage children to be creative in identifying and devising problems and solutions in all areas of learning. • Make number lines available for reference and encourage children to use them in their own play. • Big number lines may be more appropriate than counters for children with physical impairments. • Help children to understand that five fingers on each hand make a total of ten fingers altogether, or that two rows of three eggs in the box make six eggs altogether.

Additional information for the ‘exceeding’ judgement

taken from the 2014 Early Years Foundation Stage Handbook

Numbers: Children estimate a number of objects and check quantities by counting up to 20. They solve practical problems that involve combining groups of 2, 5, or 10, or sharing into equal groups.

(This descriptor has been amended to reflect the increased level of challenge applied to the expected descriptor following the Tickell review.)

Years 1 – 6, ADDITION CALCULATION and METHODS USED, Sept 2014

Year 1

Pupils should be taught to:

- read, write and interpret mathematical statements involving addition …equals (=) signs
- represent and use number … within 20
- add … one-digit and two-digit numbers to 20, including zero
- solve one-step problems that involve addition …, using concrete objects and pictorial representations, and missing number problems.

Vocabulary

Addition, add, forwards, put together, more than, total, altogether, distance between, difference between, equals = same as, most, pattern, odd, even, digit, counting on.

Some Key Questions

How many altogether? How many more to make…? I add …more. What is the total? How many more is… than…? How much more is…? One more, two more, ten more… , What can you see here?, Is this true or false?, What is the same? What is different?

Year 2

Pupils should be taught to:

Solve problems with addition…:

- using concrete objects and pictorial representations, including those involving numbers, quantities and measures
- applying their increasing knowledge of mental and written methods
- recall and use addition … facts to 20 fluently, and derive and use related facts up to 100
- add … numbers using concrete objects, pictorial representations, and mentally, including:
  - a two-digit number and ones
  - a two-digit number and tens- two two-digit numbers
  - adding three one-digit numbers
- show that addition of two numbers can be done in any order (commutative)
- recognise and use the inverse relationship between addition and subtraction and use this to check calculations and missing number problems.

Vocabulary

+, add, addition, more, plus, make, sum, total, altogether, how many more to make…? how many more is… than…? how much more is…? =, equals, sign, is the same as, Tens, ones, partition, Near multiple of 10, tens boundary, More than, one more, two more… ten more… one hundred more.

Some Key Questions

How many altogether? How many more to make…? How many more is… than…? How much more is…? , Is this true or false?

If I know that 17 + 2 = 19, what else do I know? (e.g. 2 + 17 = 19; 19 – 17 = 2; 19 – 2 = 17; 190 – 20 = 170 etc).

What do you notice? What patterns can you see?

Year 3

Pupils should be taught to:

- add … numbers mentally, including:
  - a three-digit number and ones
  - a three-digit number and tens
  - a three-digit number and hundreds
- add … numbers with up to three digits, using formal written methods of columnar addition
- estimate the answer to a calculation and use inverse operations to check answers
- solve problems, including missing number problems, using number facts, place value, and more complex addition.

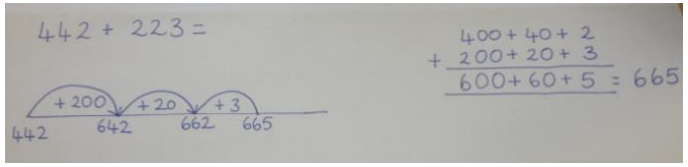
Vocabulary

Hundreds, tens, ones, estimate, partition, recombine, difference, decrease, near multiple of 10 and 100, inverse, rounding, column subtraction, exchange

See also Y1 and Y2

Key Questions

What do you notice? What patterns can you see?, When comparing two methods alongside each other: What's the same? What's different? Look at this number in the formal method; can you see where it is in the expanded method / on the number line?

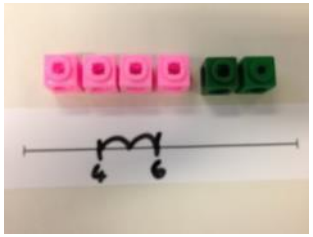


Year 4	Year 5	Year 6
Pupils should be taught to: • add … numbers with up to 4 digits using the formal written methods of columnar addition … where appropriate estimate and use inverse operations to check answers to a calculation • solve addition… two-step problems in contexts, deciding which operations and methods to use and why. **Vocabulary** add, addition, sum, more, plus, increase, sum, total, altogether, double, near double, how many more to make..? how much more? ones boundary, tens boundary, hundreds boundary, thousands boundary, tenths boundary, hundredths boundary, inverse, how many more/fewer? Equals sign, is the same as **Some Key Questions** What do you notice? What's the same? What's different? Can you convince me? How do you know?	Pupils should be taught to: • add … whole numbers with more than 4 digits, including using formal written methods (columnar addition) • add … numbers mentally with increasingly large numbers • use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy • solve addition… multi-step problems in contexts, deciding which operations and methods to use and why. **Vocabulary** tens of thousands boundary, Also see previous years **Some Key Questions** What do you notice? What's the same? What's different? Can you convince me? How do you know?	Pupils should be taught to: • perform mental calculations, including with mixed operations and large numbers. • use their knowledge of the order of operations to carry out calculations involving the four operations • solve addition… multi-step problems in contexts, deciding which operations and methods to use and why • solve problems involving addition • use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy. **Vocabulary** See previous years **Some Key Questions** What do you notice? What's the same? What's different? Can you convince me? How do you know?

STAGE 1 – (Used in sets 1 and 2)

Count on using number tracks / number lines / 100 grids to support.

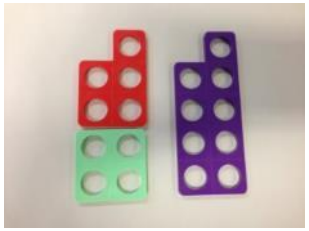




Develop concept of number bonds, initially to ten and then to 20.

Record related number facts.

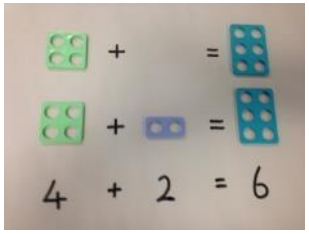
e.g. 4 +5 = 9, 5 + 4 = 9, 9 = 4 + 5, 9 = 5 + 4




STAGE 2 –(Sets 1 and 2)

Develop understanding of the equals sign / equality and the concept of 'empty box' questions.

Record solutions to calculations such as 4 + □ = 9.



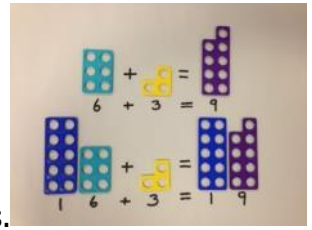
Use understanding of patterning, place value and partitioning to derive number facts. (The written recording of partitioning starts in set 2 with T & U and continues into set 4 with TTh, Th, H T and U.)

e.g. 6 + 3 = 9 (known fact)

16 + 3 = 19

26 + 3 = 29

Begin to use understanding of place value and partitioning to carry out addition of one- digit and two-digit numbers.



STAGE 3 (Set 2 to Set 4)

U + U

Continue to develop understanding of partitioning and place value and use this to support addition.

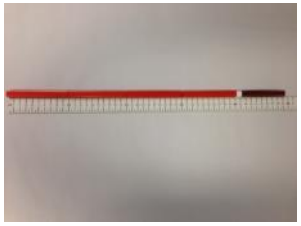
41+8

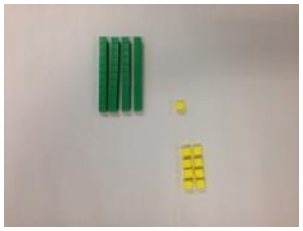
40 + 1 + 8

40 + 9= 49

Practical apparatus is used to support this, as are number tracks /100 squares and number lines.

Record the outcomes of calculations in horizontal format.





When confident with concepts of partitioning and place value, horizontal recording can be replaced with recording in columns with a focus on place value.

	T	U
	4	1
+		8
	4	9

STAGE 4

TU + TU

Continue to develop understanding of partitioning and place value and use this to support addition.

25 + 32

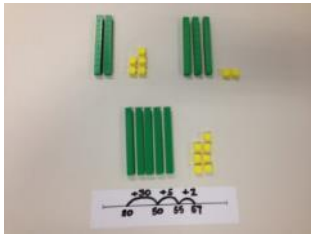
20 + 30 = 50

5 + 2 = 7

50 + 7 = 57

Practical apparatus is used to support this, as are number tracks /100 squares and number lines.

Record the outcomes of calculations in horizontal format.



Where units combine to make totals greater than 10, regroup using partitioning skills

25 + 36

20 + 30 = 50

5 + 6 = 11

50 + 11 = 50 + 10 + 1= 61

Pupils continue to determine when calculations are best carried out using mental strategies.

Horizontal recording can begin to be replaced with recording in columns with a focus on place value. Use expanded recording and apparatus to illustrate concept initially if required before moving towards the formal written method.(Expanded methods at beginning of set 4 moving to formal method by the end of the year)

	T	U
	2	5
+	3	6
	6	1

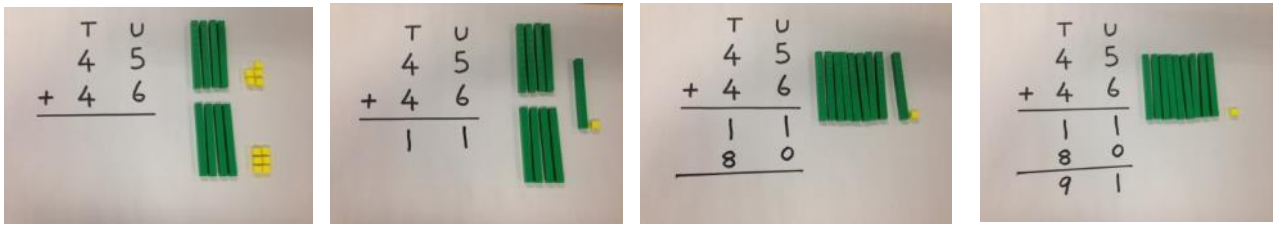
STAGE 5

Continue to determine when calculations are best carried out using mental strategies.

When written methods are more appropriate, continue use of practical apparatus to support, develop an understanding of the formal written method for column addition, initially without and then introducing carrying.

Initially use expanded recording if appropriate to explore concept.

	T	U
	4	5
+	4	6
	1	1
	8	0
	9	1



becomes

$$\begin{array}{r} T\ \ U \\ 4\ \ 5 \\ +\ 4\ \ 6 \\ \hline 9\ \ 1 \\ \hline 1 \end{array}$$

STAGE 6

Continue to determine when calculations are best carried out using mental strategies.

Extend the standard written method to introduce the hundreds column, initially without and then introducing carrying, initially using expanded recording if appropriate. (Sets 4, 5, 6 and more advanced numbers in Set 7)

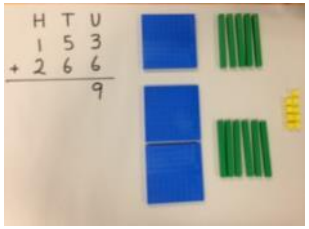
$$\begin{array}{r} H\ \ T\ \ U \\ 1\ \ 5\ \ 3 \\ +\ 2\ \ 6\ \ 6 \end{array}$$

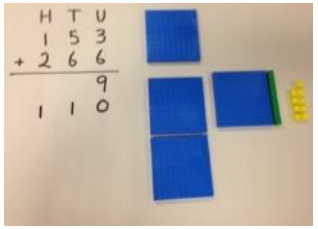
9

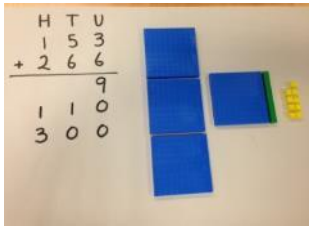
1 1 0

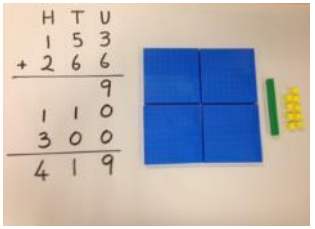
3 0 0

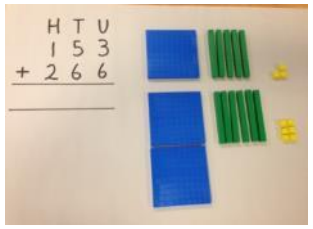
4 1 9

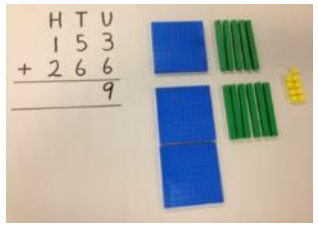


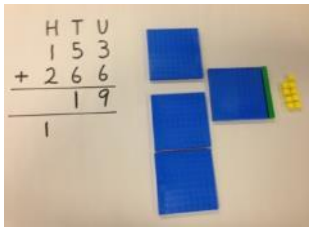


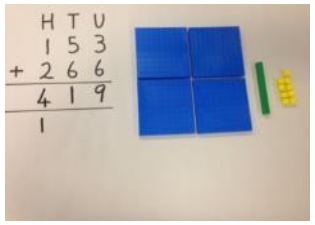












This becomes

H T U

1 5 3

+ 2 6 6

4 1 9

1

STAGE 7

Continue to determine when calculations are best carried out using mental strategies.

Develop use of the formal written method to addition of increasingly large numbers. Use expanded recording and apparatus as above to illustrate concept initially if required before moving towards the formal written method.

SUBTRACTION CALCULATION and METHODS, Sept 2014

Year 1

Pupils should be taught to:

- read, write and interpret mathematical statements involving …, subtraction (-) and equals (=) signs
- represent and use number bonds and related subtraction facts within 20
- … subtract one-digit and two-digit numbers to 20, including zero
- solve one-step problems that involve … subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = - 9.

Vocabulary

Subtraction, subtract, take away, distance between, difference between, more than, minus, less than, equals = same as, most, least, pattern, odd, even, digit,

Some Key Questions

How many more to make…? How many more is… than…? How much more is…? How many are left/left over? How many have gone? One less, two less, ten less… How many fewer is… than…? How much less is…? What can you see here? Is this true or false?

Year 2

Pupils should be taught to:

- solve problems with… subtraction:
- using concrete objects and pictorial representations, including those involving numbers, quantities and measures
- applying their increasing knowledge of mental and written methods
- recall and use … subtraction facts to 20 fluently, and derive and use related facts up to 100
- … subtract numbers using concrete objects, pictorial representations, and mentally, including:
- a two-digit number and ones
- a two-digit number and tens
- two two-digit numbers
- adding three one-digit numbers show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- recognise and use the inverse relationship between addition and subtraction and use this to check calculations and missing number problems.

Vocabulary

Subtraction, subtract, take away, difference, difference between, minus, Tens, ones, partition, Near multiple of 10, tens boundary, Less than, one less, two less… ten less… one hundred less, More, one more, two more... ten more... one hundred more

Some Key Questions

How many more to make…? How many more is… than…? How much more is…? How many are left/left over? How many fewer is… than…? How much less is…? What do you notice? What patterns can you see?

Year 3

Pupils should be taught to:

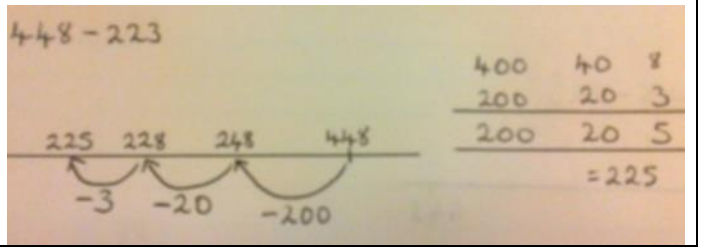
- … subtract numbers mentally, including:
- a three-digit number and ones
- a three-digit number and tens
- a three-digit number and hundreds
- … subtract numbers with up to three digits, using formal written methods of columnar … subtraction
- estimate the answer to a calculation and use inverse operations to check answers
- solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.

Vocabulary

Hundreds, tens, ones, estimate, partition, recombine, difference, decrease, near multiple of 10 and 100, inverse, rounding, column subtraction, exchange, See also Y1 and Y2

Key Questions

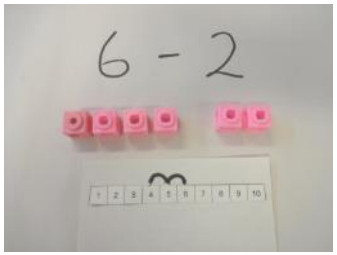
What do you notice? What patterns can you see? When comparing two methods alongside each other: What's the same? What's different? Look at this number in the formal method; can you see where it is in the expanded method / on the number line

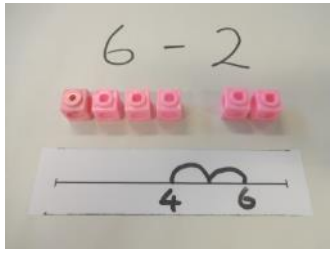


Year 4	Year 5	Year 6
Pupils should be taught to: • … subtract numbers with up to 4 digits using the formal written methods of columnar … subtraction where appropriate • estimate and use inverse operations to check answers to a calculation • solve …subtraction two-step problems in contexts, deciding which operations and methods to use and why. **Vocabulary** add, addition, sum, more, plus, increase, sum, total, altogether, double, near double, how many more to make..? how much more? ones boundary, tens boundary, hundreds boundary, thousands boundary, tenths boundary, hundredths boundary, inverse, how many more/fewer? Equals sign, is the same as. **Some Key Questions** What do you notice? What's the same? What's different? Can you convince me? How do you know?	Pupils should be taught to: • … subtract whole numbers with more than 4 digits, including using formal written methods (columnar subtraction) • … subtract numbers mentally with increasingly large numbers • use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy • solve … subtraction multi-step problems in contexts, deciding which operations and methods to use and why. **Vocabulary** tens of thousands boundary, Also see previous years **Some Key Questions** What do you notice? What's the same? What's different? Can you convince me? How do you know?	Pupils should be taught to: • perform mental calculations, including with mixed operations and large numbers. • use their knowledge of the order of operations to carry out calculations involving the four operations • solve … subtraction multi-step problems in contexts, deciding which operations and methods to use and why • solve problems involving addition, subtraction, multiplication and division • use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy. **Vocabulary** See previous years **Some Key Questions** What do you notice? What's the same? What's different? Can you convince me? How do you know?

STAGE 1 (Set 1 and 2)

Count back using number tracks / number lines / 100 grids to support the development of the concept of subtraction as take away.

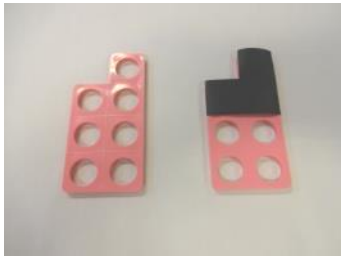




Develop subtraction facts initially to ten and then to 20.

Record related number facts (and make links to related addition facts)

e.g. 9 - 4 = 5, 9 - 5 = 4



STAGE 2 (Set 2)

Develop understanding of the equals sign / equality and the concept of 'empty box' questions.

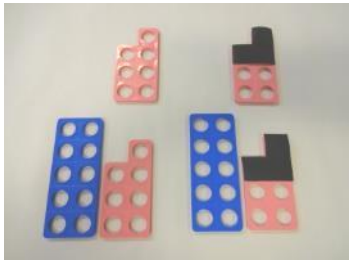
Record solutions to calculations such as 9 - = 5.

Use understanding of patterning, place value and partitioning to derive number facts.

e.g. 7 - 3 = 4 (known fact)

17 – 3 – 14

27 – 3 = 24



Begin to use understanding of place value and partitioning to support subtraction of one-digit and two-digit numbers.

STAGE 3 (Set 3)

TU + U

Continue to develop understanding of partitioning and place value and use this to support subtraction.

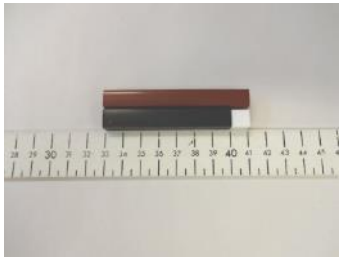
41 - 8

40 - 1 - 7

40 – 8 = 33

Practical apparatus are used to support this, as are number tracks /100 squares and number lines.

Record the outcomes of calculations in horizontal format.



41 – 8 = 33

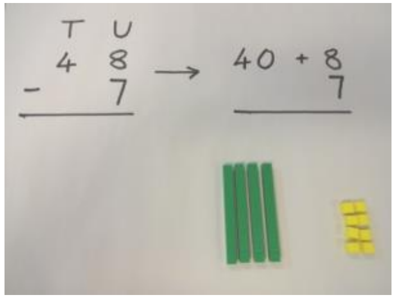
STAGE 4 (Set 4)

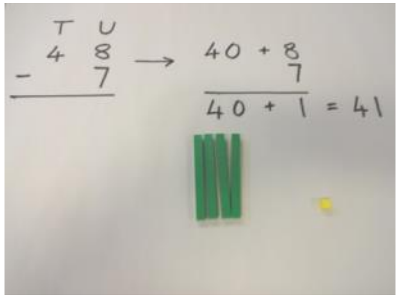
Pupils continue to continue to determine when calculations are best carried out using mental strategies.

Horizontal recording can begin to be replaced with recording in columns with a focus on place value. Use expanded recording and apparatus to illustrate concept initially if required before moving towards the formal written method.

No exchange

```
  T  U
  4  8   ------->   40 + 8
-    7            -      7
                  40 + 1 = 41
```



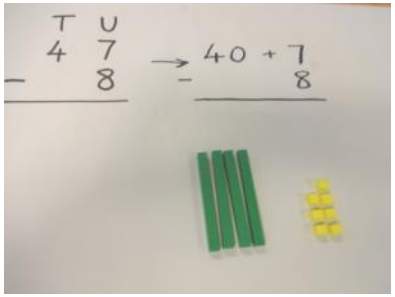


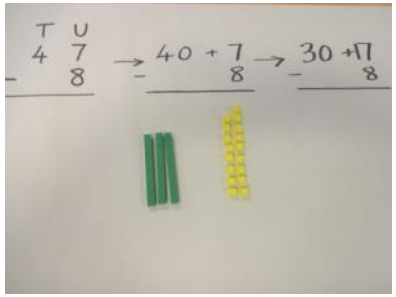
becomes

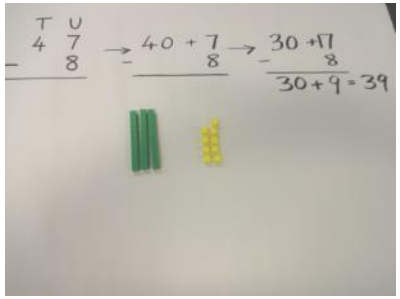
	T	U
	4	8
-		7
	4	1

Exchange

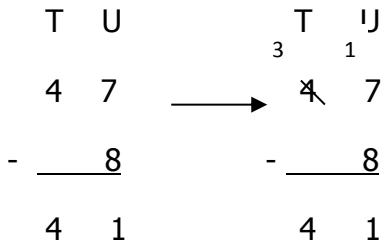
	T	U								
	4	7	⟶	4	0	+	7	⟶	3	0 + 1 7
-		8	-				8	-		8
									3	0 + 9 = 3 9







becomes



STAGE 5 (Set 4 and continues into Set 5)

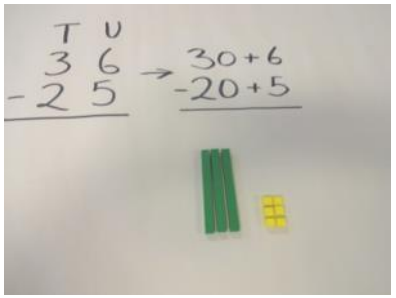
TU + TU

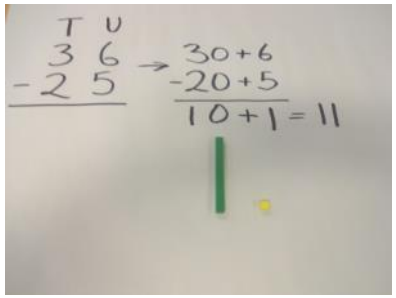
Continue to determine when calculations are best carried out using mental strategies.

Develop use of the formal written method. Use expanded recording and apparatus to illustrate concept initially if required before moving towards the formal written method.

No exchange

	T	U				
	3	6	→	30	+	6
-	2	5		- 20	+	5
				10	+	1 = 11



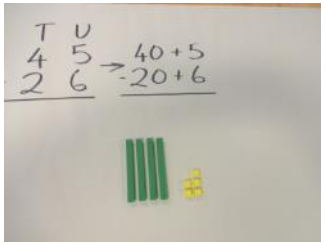


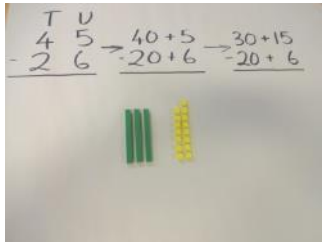
becomes

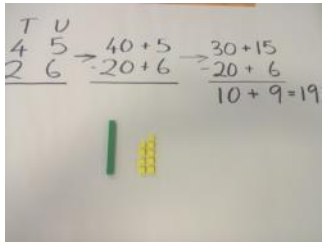
	T	U
	3	6
-	2	5
	1	1

Exchange

T	U						
4	5		40	+	5	30	+ 15
- 2	6	→	- 20	+	6	- 20	+ 6
						10	+ 9 = 19

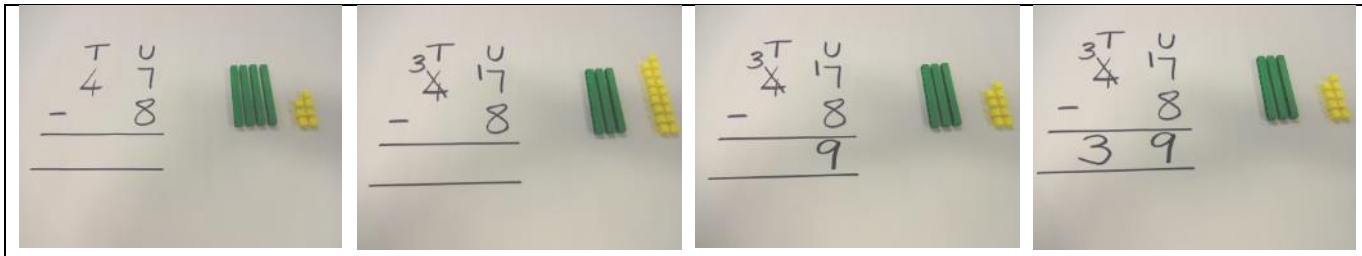






becomes

T	U					
			3		1	
4	5	→	~~4~~		5	
- 2	6		-	2	6	
			3		9	



STAGE 6 (Sets 5, 6 and 7)

HTU - HTU

Continue to determine when calculations are best carried out using mental strategies.

Develop use of the formal written method. Use expanded recording and apparatus to illustrate concept initially if required before moving towards the formal written method.

Explore how the process relates to numbers with zeros as place holders.

No exchange

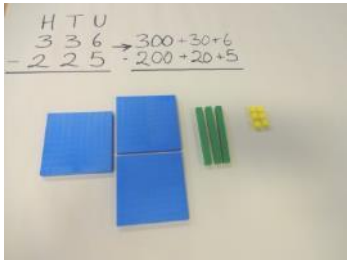
Using an expanded method of recording if appropriate before moving to formal method

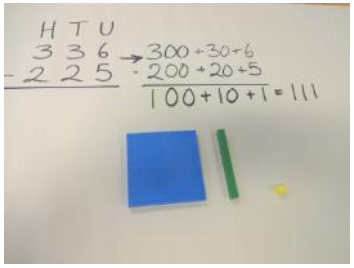
H T U

3 3 6 → 300 + 30 + 6

- 2 2 5 → - 200 + 20 + 5

100 + 10 + 1 = 111





becomes

	H	T	U
	3	3	6
-	2	2	5
	1	1	1

Exchange

H T U

4 4 5 → 400 + 40 + 5

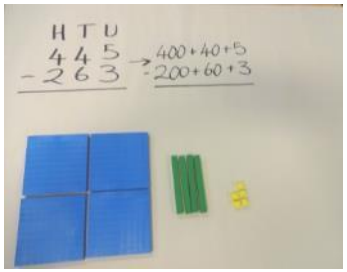
- 2 6 3 -200 + 60 + 3

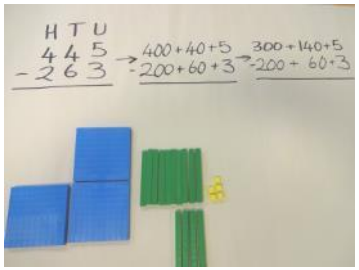
2

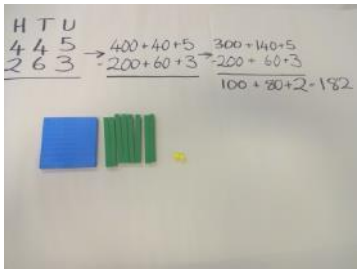
300 + 140 + 5 300 + 140 + 5

- 200 + 60 + 3 → - 200 + 60 + 3

80 + 2 100 + 80 + 2 = 182







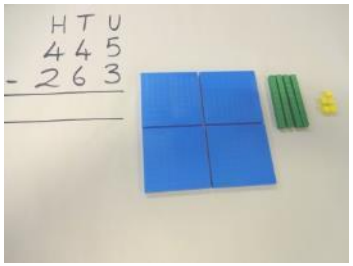
becomes

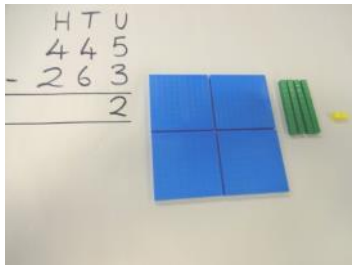
^{3}H ^{1}T U

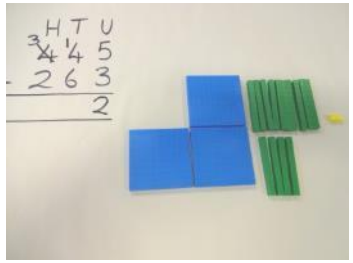
~~4~~ 4 5

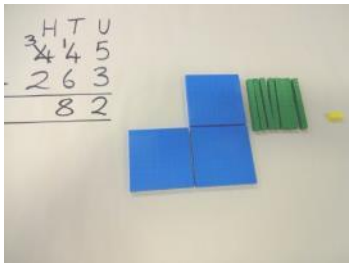
- 2 6 3

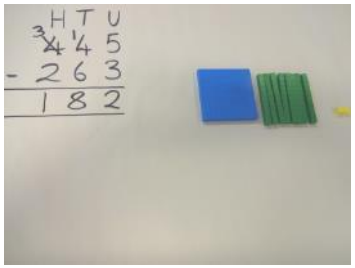
1 8 2



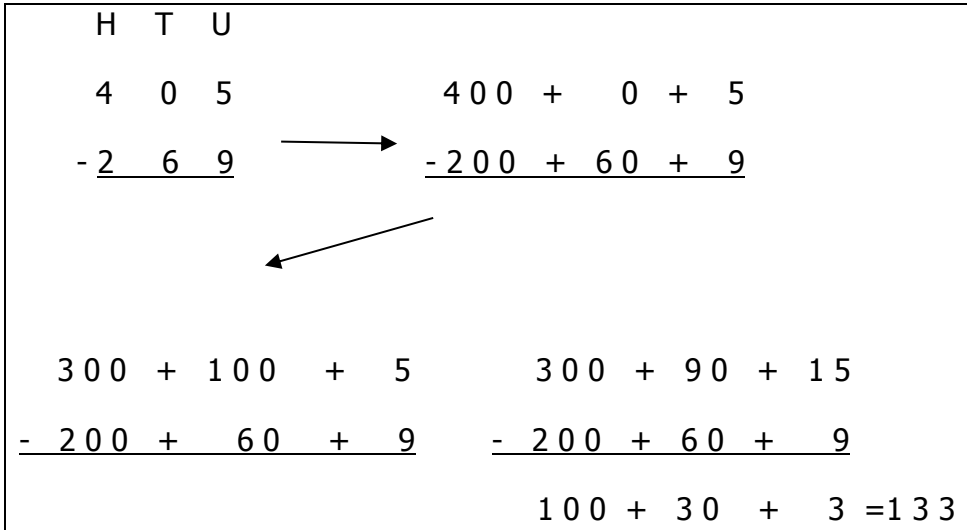




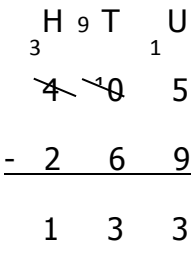




Exchange with place holders



becomes



STAGE 7 (Set 6 and 7)

Continue to determine when calculations are best carried out using mental strategies.

Develop use of the formal written method to subtraction of increasingly large numbers. Use expanded recording and apparatus as above to illustrate concept initially if required before moving towards the formal written method.

MULTIPLICATION CALCULATION and METHODS USED, Sept 2014

Year 1	Year 2	Year 3
Pupils should be taught to: • solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher. **Vocabulary** Ones, groups, lots of, doubling, repeated addition, groups of, lots of, times, columns, rows, longer, bigger, higher etc, times as (big, long, wide …etc) **Some Key Questions** Why is an even number an even number? What do you notice? What's the same? What's different? Can you convince me? How do you know?	Pupils should be taught to: • recall and use multiplication … facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers • calculate mathematical statements for multiplication … within the multiplication tables and write them using the multiplication (×), … and equals (=) signs • show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot • solve problems involving multiplication … using materials, arrays, repeated addition, mental methods, and multiplication … including problems in contexts. **Vocabulary** multiple, multiplication array, multiplication tables / facts, groups of, lots of, times, columns, rows **Some Key Questions** What do you notice? What's the same? What's different? Can you convince me? How do you know?	Pupils should be taught to: • recall and use multiplication … facts for the 3, 4 and 8 multiplication tables write and calculate mathematical statements for multiplication … using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written method solve problems, including missing number problems, involving multiplication … including integer scaling problems and correspondence problems in which n objects are connected to m objects. **Vocabulary** Partition, grid method, inverse **Some Key Questions** What do you notice? What's the same? What's different? Can you convince me? How do you know?

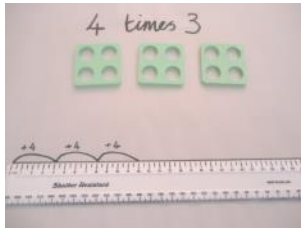
Year 4	Year 5	Year 6
Pupils should be taught to: • recall multiplication … facts for multiplication tables up to 12 × 12 • use place value, known and derived facts to multiply … mentally, including: multiplying by 0 and 1; multiplying together three numbers • recognise and use factor pairs and commutativity in mental calculations • multiply two-digit and three-digit numbers by a one-digit number using formal written layout • solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects. **Vocabulary** Factor **Some Key Questions** What do you notice? What's the same? What's different? Can you convince me? How do you know?	Pupils should be taught to: • solve problems involving multiplication and division where larger numbers are used by decomposing them into their factors • multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers multiply … numbers mentally drawing upon known facts • multiply … whole numbers and those involving decimals by 10, 100 and 1000 • solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign • solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates. **Vocabulary** cube numbers, prime numbers, square numbers, common factors, prime number, prime factors, composite numbers **Some Key Questions** What do you notice? What's the same? What's different? Can you convince me? How do you know? How do you know this is a prime number?	Pupils should be taught to: • multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication • perform mental calculations, including with mixed operations and large numbers. • use their knowledge of the order of operations to carry out calculations involving the four operations • solve problems involving addition, subtraction, multiplication and division • use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy. **Vocabulary** See previous years common factor **Some Key Questions** What do you notice? What's the same? What's different? Can you convince me? How do you know?

STAGE 1 (Set 1 and 2)

Develop multiplication as repeated grouping (repeated addition of sets of the same size) using practical apparatus and diagrams.

STAGE 2 (Set 2 and 3)

Develop an understanding of multiplication using arrays and number lines showing repeated groups.



Use number lines to show repeated grouping (repeated addition of sets of the same size).

STAGE 3 (Set 2 and 3)

Develop the use of x and = symbols to record calculations horizontally.

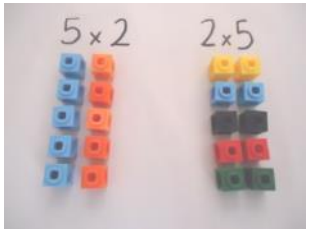
Use arrays and other practical apparatus to illustrate commutativity (that multiplication calculations can be carried out in any order) e.g. 2 x 5 arrives at the same product as 5 x 2.

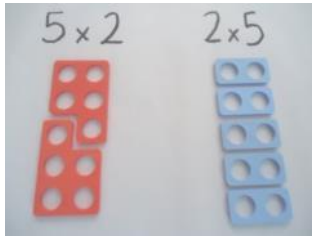
Begin to derive new facts from known facts

e.g. 3 x 2 = 6 (known fact)

30 x 2 = 60

300 x 2 = 600 etc.





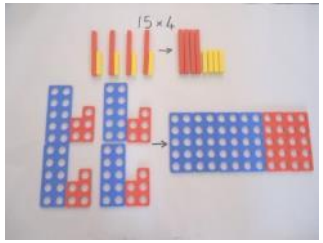
STAGE 4 (Set 3 and Set 4)

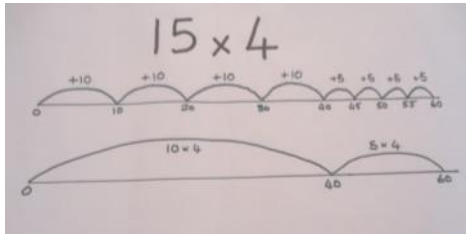
Begin to use understanding of place value and partitioning to carry out multiplication of two- digit by one -digit numbers

15 x 4

↙ ↘

10 5



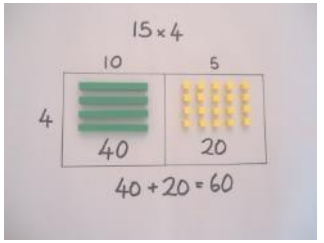


10 x 4 = 40

5 x 4 = 20

40 + 20 = 60

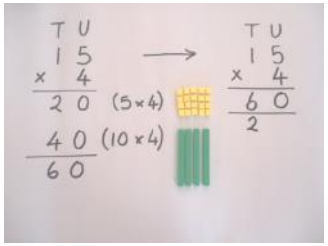
Use grid approaches to illustrate as appropriate using practical apparatus to support.



	10	5
4	40	20

Develop expanded recording in columns and then move to formal written method, using practical apparatus to support as required.

	T	U			T	U
	1	5	→		1	5
x		4		x		4
	2	0	(5 x 4)		6	0
	4	0	(10 x 4)		2	
	6	0				



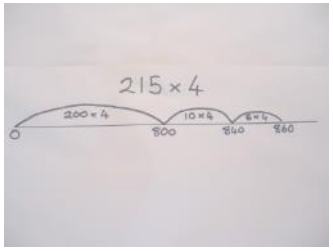
STAGE 5 (Set 5)

Extend written approaches to HTU x U, then to ThHTU x U

Illustrate using partitioning approaches as required

215 x 4

200 10 5



200 x 4 = 800

10 x 4 = 40

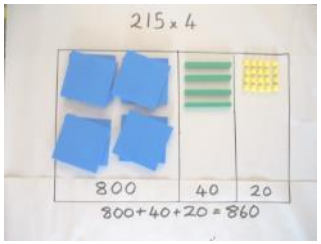
5 x 4 = 20

800 + 40 + 20 = 860

Illustrate using grid approaches as required

	200	10	5
4	800	40	20

800 + 40 + 20 = 860



Develop expanded recording in columns and then move to formal written method, using practical apparatus to support as required.

	H	T	U			H	T	U
	1	1	5	→		2	1	5
x			4		x			4
		2	0	(5 x 4)			8	6	0
		4	0	(10 x 4)				2	
	8	0	0	(200 x 4)					
	8	6							

(This type of multiplication can be taught either way; 5 x 4 or 4 x 5)

STAGE 6 (Start in Set 5 and continue into Set 6 and 7)

Extend written approaches to HTU x TU and ThHTU x TU

Illustrate using grid approaches as required

26 x 13

	20	6
10	200	60
3	60	18

200 + 60 + 60 + 18 = 338

Develop expanded recording in columns and then move to formal written method of long multiplication, using practical apparatus to support as required.

	H	T	U			H	T	U
		2	6	→			2	6
x		1	3		x		1	3
		1	8	(6 x 3)			7	8
		6	0	(20 x 3)			1	
		6	0	(6 x 10)		2	6	0
	2	0	0	(10 x 20)		3	3	8
	3	3	8			1		
	1							

226 x 13

	200	20	6
10			
3	2000	200	60
	600	60	18

2000 + 600 + 200 + 60 + 60 + 18 = 2938

Develop expanded recording in columns and then move to formal method of long multiplication, using practical apparatus to support as required.

	H	T	U		
	2	2	6		
x		1	3		
		1	8	(6 x 3)	
		6	0	(20 x 3)	
	6	0	0	(200 x 3)	
		6	0	(6 x 10)	
	2	0	0	(20 x 10)	
2	0	0	0	(200 x 10)	
2	9	3	8		
	1				

→

	H	T	U
	2	2	6
x		1	3
	6	7	8
		1	
2	2	6	0
2	9	3	8
	1		

DIVISION CALCULATION and METHODS USED, Sept 2014

Year 1

Pupils should be taught to:

- solve one-step problems involving … division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Vocabulary

share, share equally, one each, two each…, group, groups of, lots of, array

Some Key Questions

How many groups of…?

How many in each group?

Share… equally into…

What can do you notice?

Year 2

Pupils should be taught to:

- recall and use … division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- calculate mathematical statements for … division within the multiplication tables and write them using the division (÷) and equals (=) signs
- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- solve problems involving … division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

Vocabulary

group in pairs, 3s … 10s etc, equal groups of, divide, ÷, divided by, divided into, remainder

Some Key Questions

How many 10s can you subtract from 60?

I think of a number and double it. My answer is 8. What was my number?

If 12 x 2 = 24, what is 24 ÷ 2?

Questions in the context of money and measures (e.g. how many 10p coins do I need to have 60p? How many 100ml cups will I need to reach 600ml?)

Year 3

Pupils should be taught to:

- recall and use … division facts for the 3, 4 and 8 multiplication tables
- write and calculate mathematical statements for … division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- solve problems, including missing number problems, involving … division, including integer scaling problems and correspondence problems in which n objects are connected to m objects.

Vocabulary

See Y1 and Y2

inverse

Some Key Questions

Questions in the context of money and measures that involve remainders (e.g. How many lengths of 10cm can I cut from 81cm of string? You have £54. How many £10 teddies can you buy?)

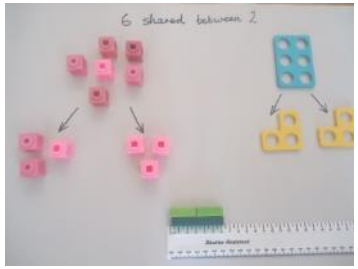
What is the missing number? $17 = 5 \times 3 + \_\_$

$\_\_ = 2 \times 8 + 1$

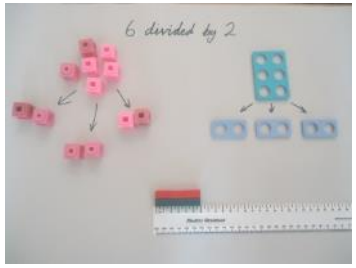
Year 4	Year 5	Year 6
Pupils should be taught to: • recall …division facts for multiplication tables up to 12 × 12 • use place value, known and derived facts to … divide mentally, including: dividing by 1; • recognise and use factor pairs and commutativity in mental calculations **Vocabulary** see years 1-3 divide, divided by, divisible by, divided into, share between, groups of, factor, factor pair, multiple, times as (big, long, wide …etc), equals, remainder, quotient, divisor, inverse **Key Questions for Year 4 to 6** What do you notice? What's the same? What's different? Can you convince me? How do you know?	Pupils should be taught to: • solve problems involving …division where larger numbers are used by decomposing them into their factors • …divide numbers mentally drawing upon known facts divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context • … divide whole numbers and those involving decimals by 10, 100 and 1000 • solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign • solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates. **Vocabulary** see year 4, common factors, prime number, prime factors, composite numbers, short division, square number, cube number, inverse, power of	Pupils should be taught to: • divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context • perform mental calculations, including with mixed operations and large numbers. • use their knowledge of the order of operations to carry out calculations involving the four operations • solve problems involving addition, subtraction, multiplication and division • use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy. **Vocabulary** see years 4 and 5

STAGE 1 (Set 1 and continued into Set 2)

Develop division as sharing

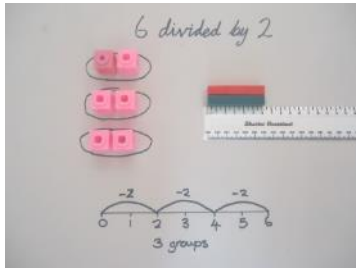


Develop division as repeated grouping (repeated subtraction of sets of the same size) using practical apparatus and diagrams.



STAGE 2 (Set 2)

Develop an understanding of division using arrays and number lines showing repeated groups Use number lines to show repeated grouping (repeated subtraction of sets of the same size)



STAGE 3 (Set 3 and 4)

Develop the use of ÷ and = symbols to record calculations horizontally

Use arrays and other practical apparatus to illustrate making of repeated groups

Begin to derive new facts from known facts

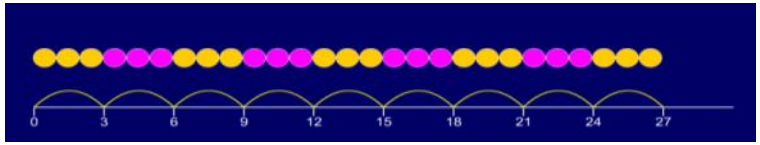
e.g. 6 ÷ 2 = 3 (known fact)

60 ÷ 2 = 30

600 ÷ 2 = 300

Begin to carry out division of two- digit by one -digit numbers, first without remainders, then introducing remainders, illustrating this using informal methods first if required.

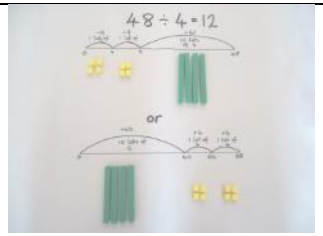
27 ÷ 3



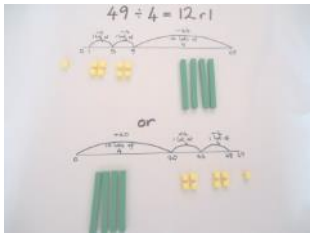
STAGE 4 (Set 3 and 4)

Division using larger multiples of the divisor, first with no remainders, then with remainders

48 ÷ 4



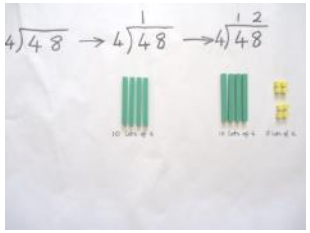
49 ÷ 4

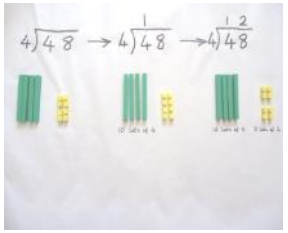


STAGE 5 (Set 4)

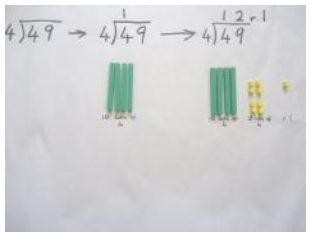
Move to develop the standard method for short division, first with no remainders, then with remainders

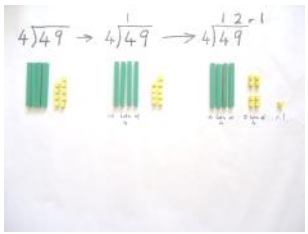
48 ÷ 4





49 ÷ 4





STAGE 6 (Last term of Set 4 into Set 5, 6 and 7)

Extend written calculation methods to HTU ÷ U, then to ThHTU ÷ U, first with no remainders, then with remainders, illustrating this using informal methods first if required.

No carrying forward required

448 ÷ 4

(as above, but with additional hundreds column)

No carrying forward required, but with remainders

449 ÷ 4

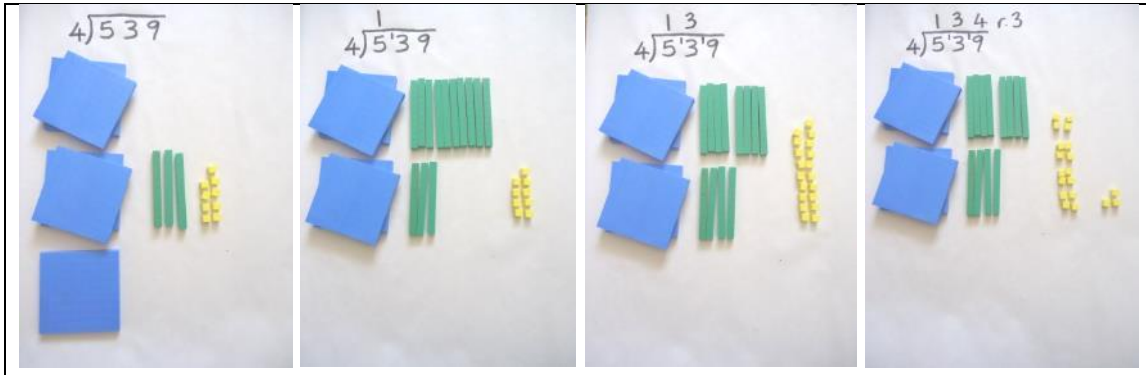
(as above, but with additional hundreds column)

Carrying forward required

536 ÷ 4

Carrying forward required, but with remainders

539 ÷ 4



STAGE 7 (Set 5, 6 and 7)

Extend written approaches to the formal method of long division when dividing by two-digit numbers, illustrating this using informal methods first if required.

```
        2  8  r 1 2
15 ) 4  3  2
     3  0  0    (15 x 20)
     1  3  2
     1  2  0    (15 x 8)
        1  2
```

becomes

```
          2  8 r 1 2
15) 4  3  2
    3  0  ↓
    1  3  2
```